

# ERROR BOUNDS FOR RAYLEIGH-RITZ MODEL REDUCTION

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Model reduction via Rayleigh-Ritz condensation is often used to create smaller models of otherwise large linear dynamic systems. The smaller models can then be used to predict efficiently the forced response of the system within a specified frequency band. Among members falling into this category is a family of Rational Krylov Methods which includes Padé Via Lanczos [1,2,3] and Forced Response Condensation [4,5]. These methods work by building a single or multi-point Padé approximation with interpolation points in the region of interest.

For practical applications, the model reduction techniques need to be complemented by error bounds to ensure that the reduced model retains sufficiently high accuracy to meet the purposes of the analysis. We present a scheme to compute error bounds for a class of Ritz reduction strategies. We divide the problem of bounding the error into two parts: determining that each “resonant” eigenpair in the exact response is accurately represented in the reduced model, and bounding the nonresonant contributions. In this context, we bound the resonant contribution of the error by ensuring that the contribution of each eigenpair in the frequency band of interest is estimated by a corresponding Ritz pair. This property is verified by using Sylvester’s theorem in the case of undamped systems. We also present bounds for the nonresonant contribution to the error. By combining these results we obtain overall upper and lower bounds on the reduction error response.

We compare the proposed error bound to existing error estimates for Krylov Reduction Methods based on their particular implementations.

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## References

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